

# Ergodic Theory of Dynamical Systems

**Objectives:** The presentation of some basic results in ergodic theory and their relationship with number theory and dynamical systems theory.

**Prerequisites:** Special topics in functional analysis, real and complex analysis, probability theory.

## Syllabus

1. **Dynamical systems with an invariant measure.** Poincaré's recurrence. The Bogoliubov-Krylov theorem.
2. **The main ergodic theorems.** Uniform distribution (mod 1) and Weyl's ergodic theorem. The ergodic theorems of von Neumann and Birkhoff. Ergodic maps.
3. **Mixing.** Ergodicity. Weak mixing. Strong mixing. Markov and Bernoulli shifts.
4. **Special classes of mappings.** Piecewise monotonic mappings. Denjoy diffeomorphisms. Billiards.
5. **Entropy.** Metric entropy. Topological entropy. The variational principle.
6. **Recurrence and its applications to combinatorics.** The theorems of van der Waerden and Szemerédi.
7. **Applications to chaotic dynamical systems.** The ergodic theorem of Oseledec. Liapunov exponents. Strange attractors.

## Bibliography

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7. M. Viana, *Dynamics: A Probabilistic and Geometric Perspective*, Proceedings of the International Congress of Mathematicians, Vol. I (Berlin, 1998). Doc. Math. 1998, Extra Vol. I, 557-578 (electronic).
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9. K. Yosida, *Functional Analysis*, 5<sup>th</sup> ed., Springer-Verlag, Berlin, 1995.

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